I'm not robot

reCAPTCHA

Continue

Exponential growth and decay word problems calculator answers key pdf

You should choose the time unit in a way that corresponds to the nature of the observed process. So, the projected number of inhabitants of our small city in the year 2030 is around 17,103. What should you do to calculate the projected population size in the year 2030? I plug these values into the formula, and then I simplify to find A: A = 100e36(ln(2)/6.5) = 4647.75313957... Let's do it step by step: Insert x(t) = 30,000 into the formula: 30,000 = 10,000 * 1.05t After dividing both sides of the equation by 10,000, we get: 1.05t = 3. The reason for this is that you cannot have a decline of more than 100% with regards to the initial quantity, as it would result in a negative value. Log ProbsExpo Decay Exponential word problems almost always work off the growth / decay formula, A = Pert, where "A" is the ending amount of whatever you're dealing with (for example, money sitting in an investment, bacteria growing in a petri dish, or radioactive decay of an element highlighting your X-ray), "P" is the beginning amount of that same "whatever", "r" is the growth or decay rate, and "t" is time. So I'll plug all the known values into the exponential-growth formula, and then solve for the growth constant: A = Pekt 450 = 100e6k 4.5 = e6k ln(4.5) = 6k ln(4.5)/6 = k = 0.250679566129... And did you know that... In the above case, this would start by noting that "a day and a half" is 36 hours, so we have: Use this as the power on 2: 100 × 2(72/13) = 4647.75314... (Round k to two decimal places.) For this exercise, the units on time t will be hours, because the growth is being measured in terms of hours. A more realistic model of population growth is the logistic growth model, which has the carrying capacity, a constant representing the population's natural growth limit. For some problems, these will be seconds, for others, years. So, in this example we have x(6) = 1/2 * 95 mg = 47.5 mg. For a given initial quantity of radioactive substance, you may write down the law which governs its decay over time. The growth constant is 0.25/hour. Six hours later, he measures 450 bacteria. Here, it will be easier to use the alternative notation for the exponential growth formula: x(t) = 95 * ek*t. If the bacteria doubled every six hours, then there would be 200 in six hours, 400 in twelve hours, 800 in eighteen hours, 1600 in twenty-four hours, 3200 in thirty hours, and 6400 in thirty-six hours. You can do a rough check of this answer, using the fact that exponential processes involve doubling (or halving) times. Get comfortable with this formula; you'll be seeing a lot of it. In 36 hours, there will be about 4648 bacteria. And, yes, you'd use a base of 3 if you'd been given a tripling-time, a base of 4 for a quadrupling-time, etc. So, for now, the growth constant will remain this "exact" value. At time t = 0 hours, he puts one hundred bacteria into what he has determined to be a favorable growth medium. To learn more, please see our compound interest calculator. Since the base of this exponential function is 1.05, and since it is greater than 1, the exponential growth graph we get is rising. Since 10 pm is ten hours later than noon, we want to know the amount of caffeine at t = 10. From the given data, we can conclude the initial population value, x0, equals 10,000. Here t is the number of years passed since 2019. So this exercise actually has two unknowns, the growth constant k and the ending amount A. If I had come up with a negative value for the growth constant, then I would have known to check my work to find my error(s). The answer would therefore be x(-19) = 10,000 * 1.05-19 = 3.957 inhabitants, as you can also see on this graph: Example of exponential growth graph - population size For some applications, for example when calculating the exponential decay of a radioactive substance, an alternative way of writing down the formula for exponential growth and decay is more productive: x(t) = x0 * ek*t. And why do they tell me what the doubling time is? However, this does not prevent us from using this formula with negative time values. For more examples of where you can use this formula, please check below. In our case, for the year 2030, we should use t = 11, since this is the difference in the number of years between 2030 and the initial year 2019. The data from the table are all points lying on the continuous graph of the exponential growth function: x(t) = 10,000 * 1.05t. URL: There is a substantial number of processes for which you can use this exponential growth calculator. If the initial population is 100, then, in 6.5 hours (being the specified doubling time), the population will be 200. A certain type of bacteria, given a favorable growth medium, doubles in population every 6.5 hours. (I might want to check this value quickly in my calculator, to make sure that this growth constant is positive, as it should be. Exponential growth occurs by multiplying the initial value by some constant factor at each time step. Linear growth means we add the same amount at each time step. So, at 10 pm, the amount of caffeine remaining in your body will be approximately 30 mg. Here we know how much is x(t), but we don't know the value of t when this will happen. Note that the constant was positive, because it was a growth constant. The exponential function appearing in the above formula has a base equal to 1 + r/100. I know that P = 100, and I need to find A at t = 36. If you're required to use the first method for every exercise of this type, then do so (in order to get the full points). The general rule of thumb is that the exponential growth formula: x(t) = x0 * (1 + r/100)t is used when there is a quantity with an initial value, x0, that changes over time, t, with a constant rate of change, r. Exponential Growth and Decay The exponential-growth/-decay formula A = Pert is related to the compound-interest formula, and represents the case of the interest being compounded "continuously". However, if you see this topic again in chemistry or physics, you will probably be expected to use proper units ("growth-decay constant / time"), as I have displayed above. The exponential growth equation is used in radiocarbon dating, PCR (you can discover why with our annealing temperature calculator) as well as in calculating compound interest. In real life situations there are natural oscillations of the rate of growth which are not included in this model of exponential growth. If I have a negative value at this stage, I need to go back and check my work.) Now that I have the growth constant, I can answer the actual question, which was "How many bacteria will there be in thirty-six hours?" This means using 100 for P, 36 for t, and the above expression for k. In some cases, the variable which measures the rate of change can be different than time. Therefore, the exponential growth formula we should use is: x(t) = 10,000 * (1 + 0.05)t = 10,000 * 1.05t. To solve this, you would use t = -19, since the year 2000 precedes the year 2019 by 19 years. Half-life is defined as the time needed a given quantity to reduce to half of its initial value. Not all algebra classes cover this method. On the other hand, if you're going to calculate the amount of coffee remaining in your body after you drank a cup of it, the appropriate time unit should be hours or maybe minutes. In this problem, I know that time t will be in hours, because they gave me growth in terms of hours. Many math classes, math books, and math instructors leave off the units for the growth and decay rates. We will use the fact that the half-life of caffeine in the human body is approximately six hours. A biologist is researching a newly-discovered species of bacteria. Consider the following problem: the population of a small city at the beginning of 2019 was 10,000 people. They gave me the doubling time because I can use this to find the growth constant k. Given that there were approximately 100 bacteria to start with, how many bacteria will there be in a day and a half? This calculation results in the following table, where we round the results to the nearest integer: year t x(t) 2019 0 10,000 2020 1 10,500 2021 2 11,025 2022 3 11,576 2023 4 12,155 2024 5 12,763 2025 6 13,401 2026 7 14,071 2027 8 14,775 2028 9 15,513 2029 10 16,289 2030 11 17,103 If you want to get an even better feel for the population growth, you may represent this data graphically, with the horizontal axis being the time axis and the vertical axis representing the population value x(t). Take the logarithm to the base 1.05 of both sides of this equation: t = log1.05 3. Here is the step by step calculation: Insert x(6)= 47.5 and t = 6 into the equation: 47.5 = 95 * e6k. Please note that t doesn't really have to be considered as time only. Radioactive decay is a well-known example of where the exponential decay formula is used. Therefore, the exponential decay formula in our example is: x(t) = 95 * e-0.1155 * t. Warning: When doing the above simplification of 100e36(ln(2)/6.5), try to do the calculations completely within your calculator in order to avoid round-off error. Exponential decay is given by the formula: $X_t = X_0 \times \exp(\mu t)$ where X_t is the quantity at time t, X_0 is the initial quantity, and μ is the decay constant. The ending amount is A = 450 at t = 6 hours. For example, when studying the way that atmospheric pressure changes with altitude, the variable measuring this change is distance, and you should choose meters as the appropriate units of change. If the bacteria doubled every seven hours, then there would be 200 in seven hours, 400 in fourteen hours, 800 in twenty-one hours, 1600 in twenty-eight hours, and 3200 in thirty-five hours. The answer we got above, 4678 in thirty-six hours, fits nicely between these two estimates. Let us start with x0 = 100 and, using the exponential growth calculator, see what x(10) will be for four different values of r: r x0 x(10) 1% 100 110.5 3% 100 134.4 5% 100 162.9 10% 100 259.4 From this table, we see that all initial values are the same, being equal to x0 = 100, but the final values of x(10) differ significantly. It's not a bad idea to get into the habit now of checking and reporting your units. So I'll convert "a day and a half" to "thirty-six hours", so my units match. Continuing with our small city, the next question you may ask yourself is "when can we expect the population to reach some important value?" This is useful if you want to know when to adjust the city's urban planning for a larger population, so the city council needs to know which year they can expect the city's population to have tripled in size from the original 10,000? I can use the doubling time to find the growth constant, at which point the only remaining value will be the ending amount, which is what they actually asked for. For example, if you want to understand the change in the population of a city, you should choose years. The coefficient k plays the role of the rate of growth, similarly as r does in the original exponential growth formula. Your intuition may trick you here because the difference between 1% and 3% doesn't look like much, but after ten periods, this amounts to a 21.67% higher value for x(10) for 3%-growth as compared to 1%-growth. Assuming exponential growth, what is the growth constant "k" for the bacteria? But, maybe a more fun example is to measure how much coffee remains in your body at 10 pm if you drank a cup of coffee with x0 = 95 mg of caffeine at noon. Time can be expressed in basically any appropriate units. It is best to work from the inside out, starting with the exponent, then the exponential, and finally the multiplication, like this: Note: When you are given a nice, neat doubling time, another method for solving the exercise is to use a base of 2. In the case of population growth, you may ask the question: what was the population of our small city in the year 2000, assuming the population growth rate was a constant 5%? The only variable I don't have a value for is the growth constant k, which also happens to be what I'm looking for. It was noticed that the population of the city grows at a steady rate of 5% annually. First, figure out how many doubling-times that you've been given. For instance, all of the following represent the same relationship: A = Pert A = Pekt Q = Nekt Q = Q0ekt No matter the particular letters used, the green variable stands for the ending amount, the blue variable stands for the beginning amount, the red variable stands for the growth or decay constant, and the purple variable stands for time. So first I'll find the constant. This means that we describe the phenomenon of interest in the time before the initial observation was made. The difference in the exponential growth rate r will have a significant influence on how quickly the observed quantity changes from the initial value. This expression, after dividing both sides of the equation by 95 and applying the natural logarithm, gives: 6*k = ln 0.5. By using the natural logarithm calculator, we get: k = -0.1155. Then, once I have this constant, I can go on to answer the actual question. Note that the exponential growth rate, r, can be any positive number, but, this calculator also works as an exponential decay calculator - where r also represents the rate of decay, which should be between 0 & -100%. But what is the growth constant k? So, the answer to the council's question is approximately 22 years after the initial year of 2019, so in 2041: Example of exponential growth graph - population size You may have already noticed a problem with exponential growth and decay, that it naturally treats time as only a positive value, so we are predicting a future quantity. If you compare the 10%-growth to 5%-growth, you will notice an even greater difference, 59.23% in favor of 10%-growth. I'll set this up and solve for k: A = Pekt 200 = 100e6.5k 2 = e6.5k At this point, I need to use logs to solve: ln(2) = 6.5k ln(2)/6.5 = k I could simplify this to a decimal approximation, but I won't, because I don't want to introduce round-off error if I can avoid it. Namely, it is hard to expect that the yearly rate of growth for the city's population would remain at 5% for a decade or more. you can verify if a set of numbers obeys the exponential growth formula by using the well-known Benford's law? How Populations Grow: The Exponential and Logistic Equations Two Models of Population Growth The World's Population Hasn't Grown Exponentially for at Least Half a Century Exponential Population Growth Caffeine Pharmacology Caffeine Effects on Sleep Taken 0, 3, or 6 Hours before Going to Bed Exponential growth is described by the formula: $X_t = X_0 \times (1 + r/100)^t$ where X_t is the quantity at time t, X_0 is the initial value, r is the rate of change. If you want to dig a bit deeper into this particular formula, you can use our exponential growth calculator to find out the projected number of inhabitants for each year, starting from 2019. We have: x(10) = 95 * e-0.1155 * 10 = 29.9305. The beginning amount P is the amount at time t = 0, so, for this problem, P = 100. The doubling time in this case is 6.5 hours, or between 6 and 7 hours. Finally, we get: x(11) = 10,000 * 1.0511 = 17,103. Also, we have the growth rate of r = 5%. Note that the variables may change from one problem to another, or from one context to another, but that the structure of the equation is always the same. You can observe this contrast in the following graphical representation of the four exponential growth functions: Comparison of exponential growths with different rates of growth The formula for exponential growth and decay is used to model various real-world phenomena: population growth of bacteria, viruses, plants, animals and people decay of radioactive matter blood concentration of drugs atmospheric pressure of air at a certain height compound interest and economic growth radiocarbon dating processing power of computers etc. Comparing the above equation with the original one, you can see that the relation between r and k is as follows: 1 + r/100 = ek, which means r = 100 * (ek - 1) and k = ln(1 + r/100). Use the logarithm calculator to finally get: t = 22.52. Otherwise, this base-2 trick can be a time-saver. The main difference between this graph and the normal exponential function graph is that its y-intercept is not 1 but 10,000, which corresponds to the initial value x0: Example of exponential growth graph - population size From this example, we can see the possible limitations of the exponential growth model - it is unrealistic for the rate of growth to remain constant over time.